#### CS11-747 Neural Networks for NLP

## Models w/ Latent Random Variables

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Site <a href="https://phontron.com/class/nn4nlp2019/">https://phontron.com/class/nn4nlp2019/</a>

Slides from Graham Neubig

## Discriminative vs. Generative Models

- Discriminative model: calculate the probability of output given input P(Y|X)
- Generative model: calculate the probability of a variable P(X), or multiple variables P(X,Y)
- Which of the following models are discriminative vs. generative?
  - Standard BiLSTM POS tagger
  - Globally normalized CRF POS tagger
  - Language model

### Types of Variables

- Observed vs. Latent:
  - Observed: something that we can see from our data, e.g. X or Y
  - Latent: a variable that we assume exists, but we aren't given the value
- Deterministic vs. Random:
  - Deterministic: variables that are calculated directly according to some deterministic function
  - Random (stochastic): variables that obey a probability distribution, and may take any of several (or infinite) values

## Quiz: What Types of Variables?

- In the an attentional sequence-to-sequence model using MLE/teacher forcing, are the following variables observed or latent? deterministic or random?
  - The input word ids f
  - The encoder hidden states h
  - The attention values a
  - The output word ids e

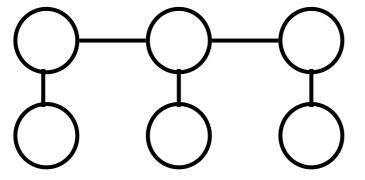
# Goal of Latent Random Variable Modeling

 Specify structural relationships in the context of unknown variables, to learn interpretable structure

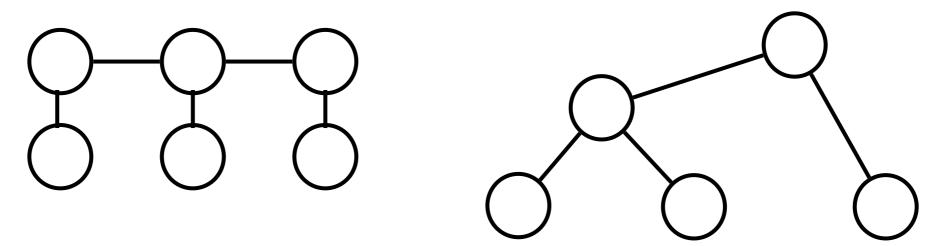
Inject inductive bias / prior knowledge

- Older latent variable models
  - Topic models (unsupervised)

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  - Hidden Markov Model (unsupervised tagger)



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  - Topic models
  - Hidden Markov Model (unsupervised tagger)



Some tree-structured Model (unsupervised parsing)

### Why Latent Random Variable

Specify structure, but interpretable structure is often discrete

 There is always a tradeoff between interpretability and flexibility

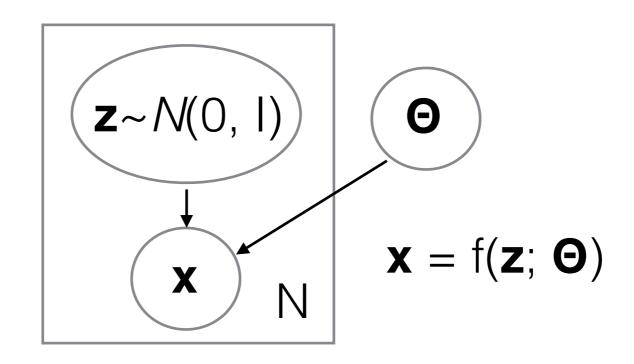
- Deep latent variable models
  - Variational Autoencoders (VAEs)
  - Generative Adversarial Network (GANs)
  - Flow-based generative models

### Variational Auto-encoders

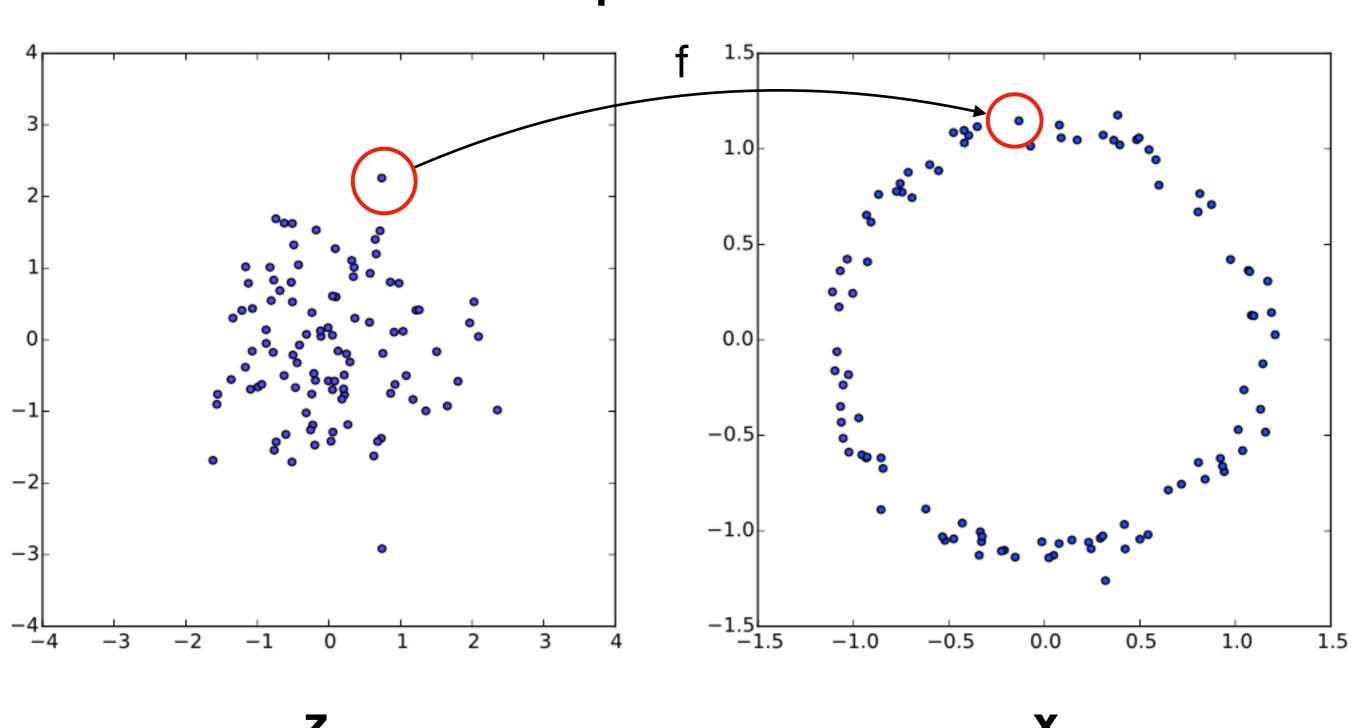
(Kingma and Welling 2014)

#### A Latent Variable Model

- We observed output x (assume a continuous vector for now)
- We have a latent variable z generated from a Gaussian
- We have a function f, parameterized by Θ that maps from z
  to x, where this function is usually a neural net

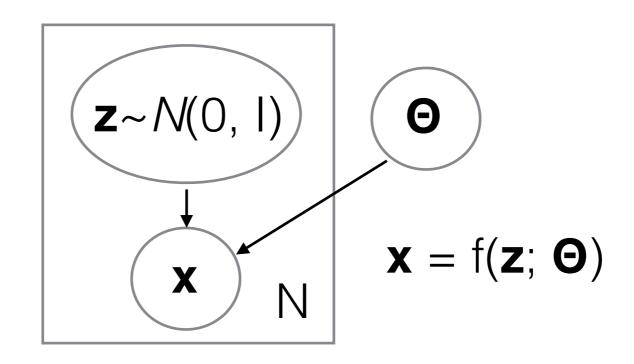


### An Example (Goersch 2016)



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#### What is Our Loss Function?

We would like to maximize the corpus log likelihood

$$\log P(\mathcal{X}) = \sum_{\boldsymbol{x} \in \mathcal{X}} \log P(\boldsymbol{x}; \theta)$$

For a single example, the marginal likelihood is

$$P(\boldsymbol{x}; \theta) = \int P(\boldsymbol{x} \mid \boldsymbol{z}; \theta) P(\boldsymbol{z}) d\boldsymbol{z}$$

We can approximate this by sampling zs then summing

$$P(\boldsymbol{x}; \theta) pprox \sum_{\boldsymbol{z} \in S(\boldsymbol{x})} P(\boldsymbol{x}|\boldsymbol{z}; \theta)$$
 where  $S(\boldsymbol{x}) := \{\boldsymbol{z}'; \boldsymbol{z}' \sim P(\boldsymbol{z})\}$ 

#### Variational Inference

$$\log p_{\boldsymbol{\theta}}(\mathbf{x}) \geq \text{ELBO}$$

$$\underbrace{\mathbb{E}_{\mathbf{z} \sim q_{\phi}(\mathbf{z}|\mathbf{x})}[\log p_{\theta}(\mathbf{x}|\mathbf{z})]}_{\text{Reconstruction Loss}} - \underbrace{D_{\text{KL}}(q_{\phi}(\mathbf{z}|\mathbf{x})||p(\mathbf{z}))}_{\text{KL Regularizer}}$$

The inequality holds for any q (z|x), but the lower bound is tight only if q(z|x) = p(z|x)

p(z|x) is intractable

#### Practice

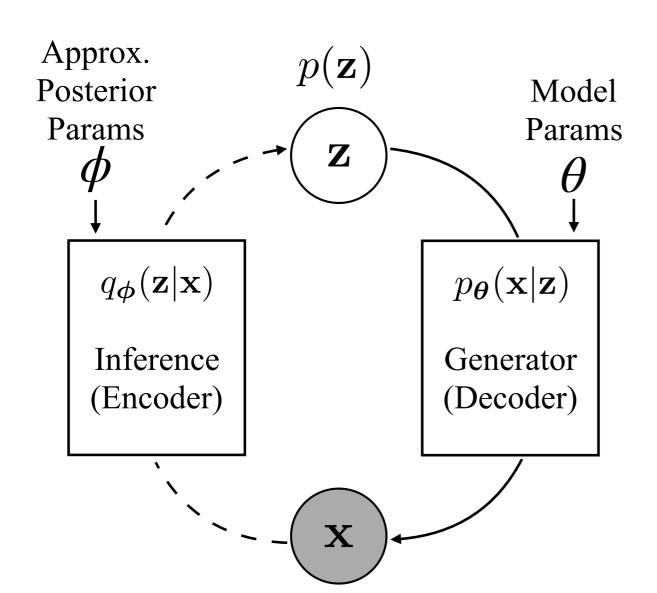
Prove

$$\log p_{\boldsymbol{\theta}}(\mathbf{x}) >= \underbrace{\mathbb{E}_{\mathbf{z} \sim q_{\boldsymbol{\phi}}(\mathbf{z}|\mathbf{x})}[\log p_{\boldsymbol{\theta}}(\mathbf{x}|\mathbf{z})]}_{\text{Reconstruction Loss}} - \underbrace{D_{\text{KL}}(q_{\boldsymbol{\phi}}(\mathbf{z}|\mathbf{x})||p(\mathbf{z}))}_{\text{KL Regularizer}}$$

Hint: use Jensen's inequality

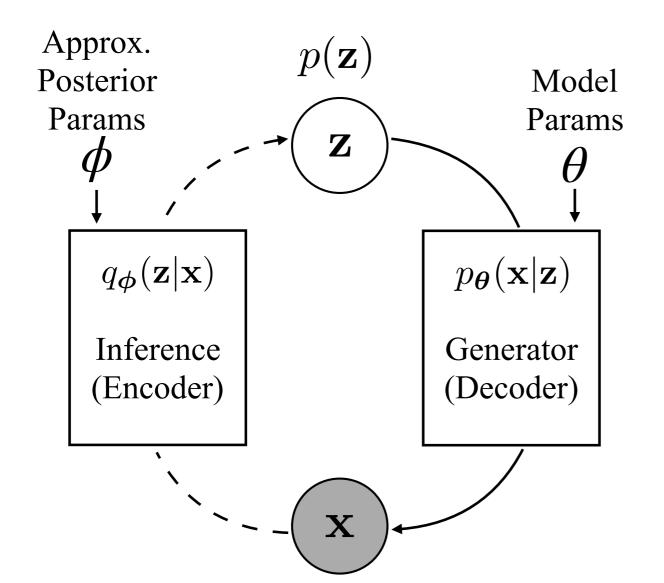
#### Variational Autoencoders

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#### Variational Autoencoders

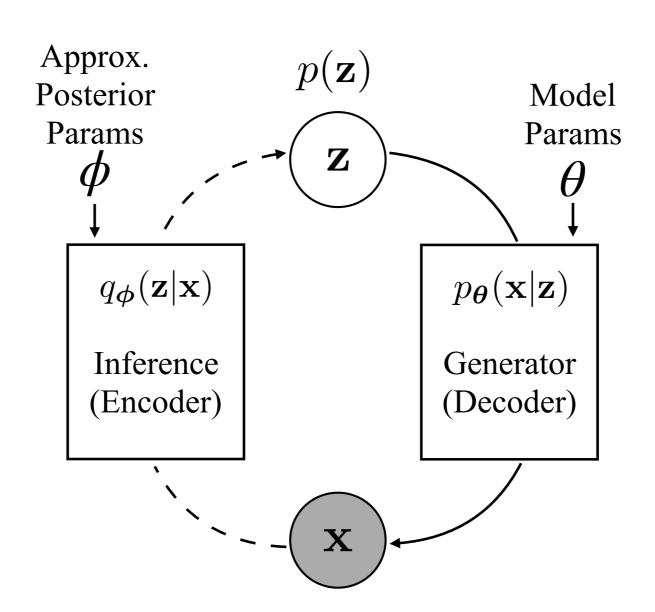
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Regularized Autoencoder

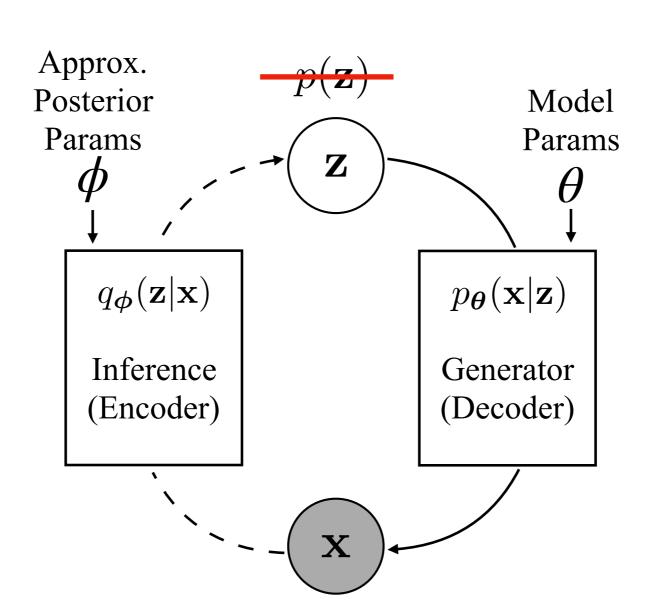
## Why prior?

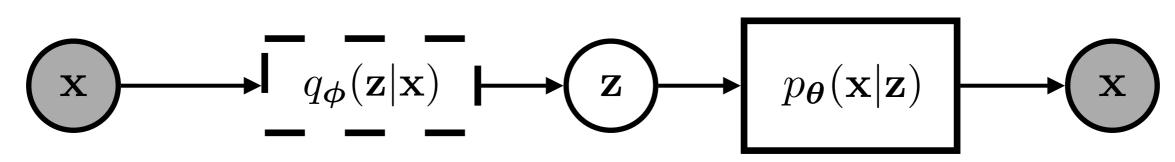
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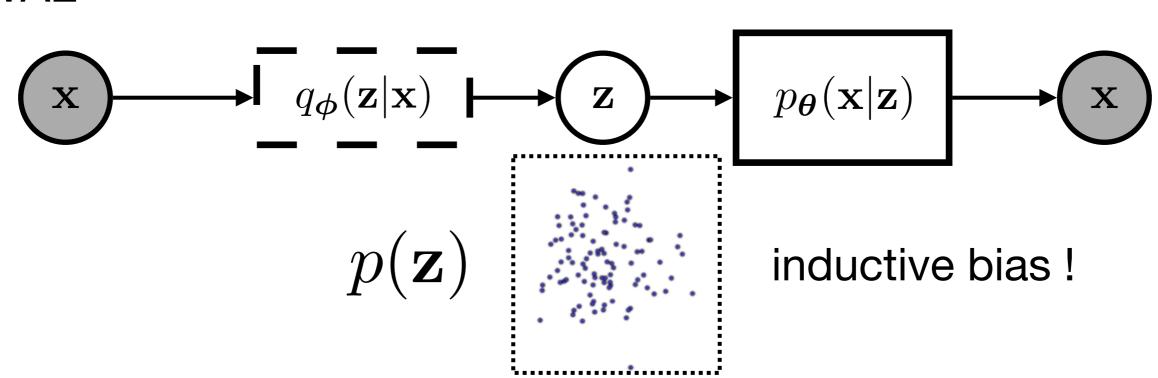


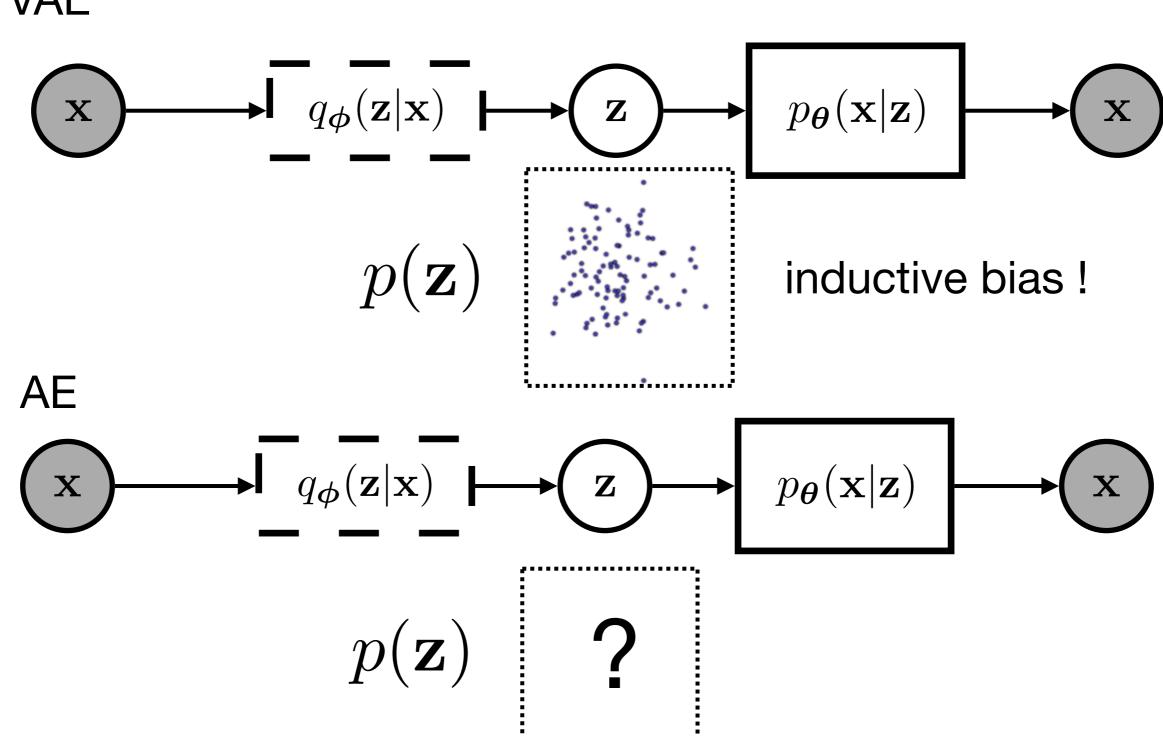
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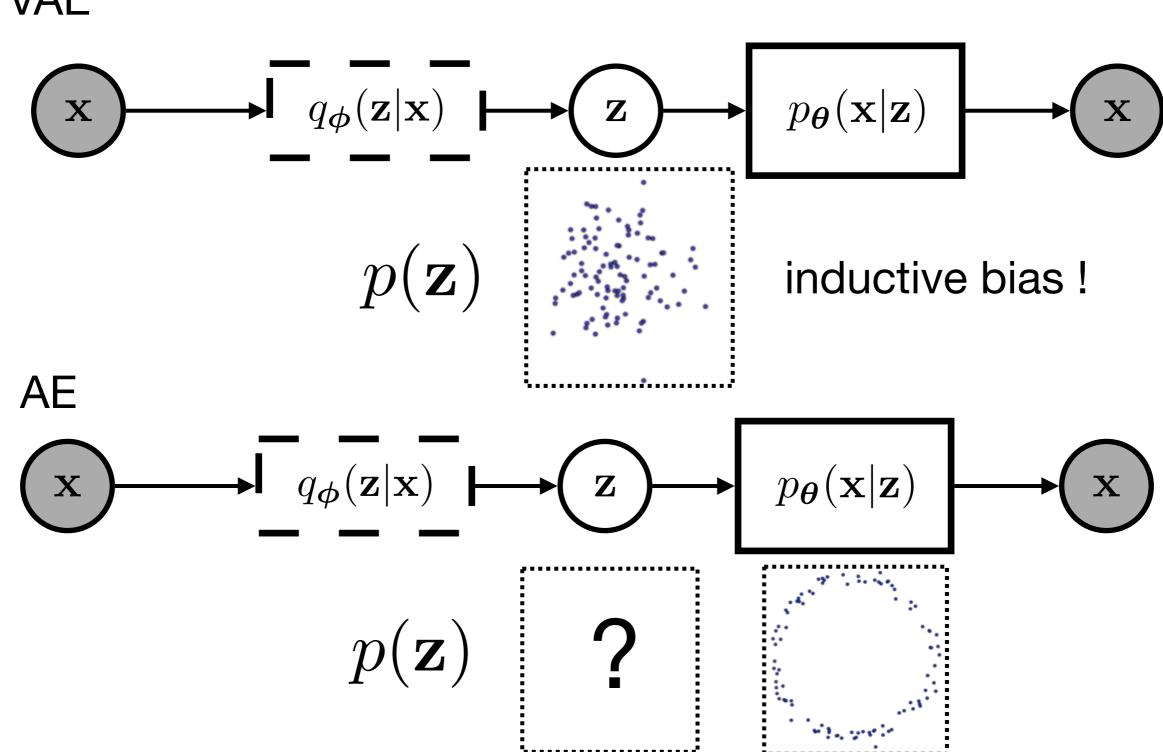
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- Generative modeling
- Representation learning
  - Representation space can be regularized by prior
- unsupervised learning

	VAE	AE	
Generative modeling	Yes	No	
Representation Learning	Yes	Yes	
Unsupervised Learning	Yes	Yes	
Controlled representation space	Yes	No	

	VAE	AE	LSTM LM	
Generative modeling	Yes	No		
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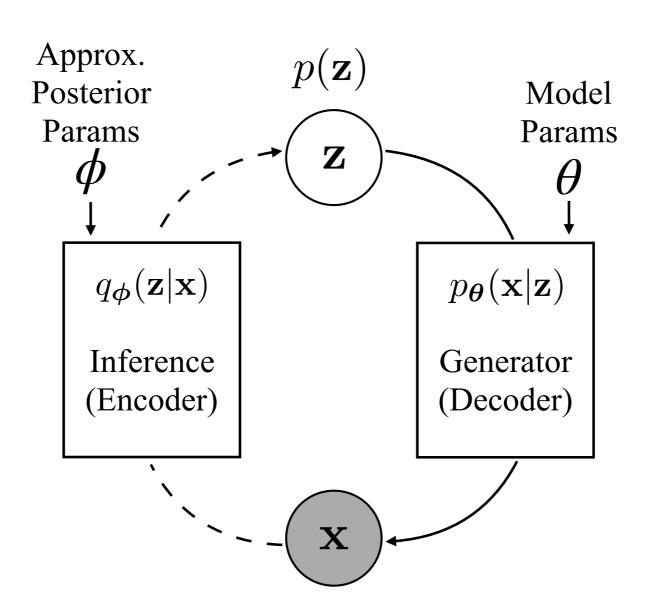
	VAE	ΑE	LSTM LM	CNN Classifier
Generative modeling	Yes	No	Yes	
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## Learning VAE

$$\log p_{\boldsymbol{\theta}}(\mathbf{x}) >= \underbrace{\mathbb{E}_{\mathbf{z} \sim q_{\boldsymbol{\phi}}(\mathbf{z}|\mathbf{x})}[\log p_{\boldsymbol{\theta}}(\mathbf{x}|\mathbf{z})]}_{\text{Reconstruction Loss}} - \underbrace{D_{\text{KL}}(q_{\boldsymbol{\phi}}(\mathbf{z}|\mathbf{x})||p(\mathbf{z}))}_{\text{KL Regularizer}}$$



## Problem! Sampling Breaks Backprop

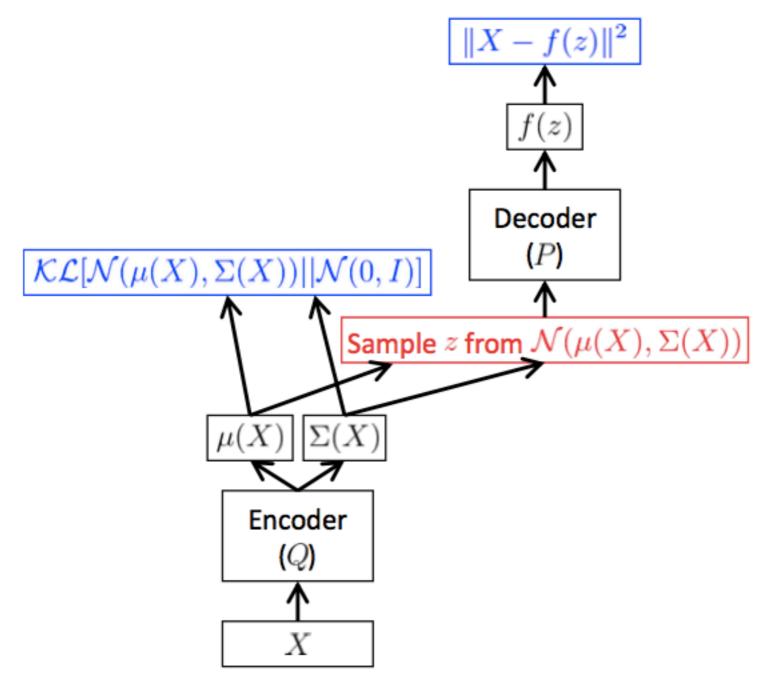


Figure Credit: Doersch (2016)

### Solution: Re-parameterization Trick

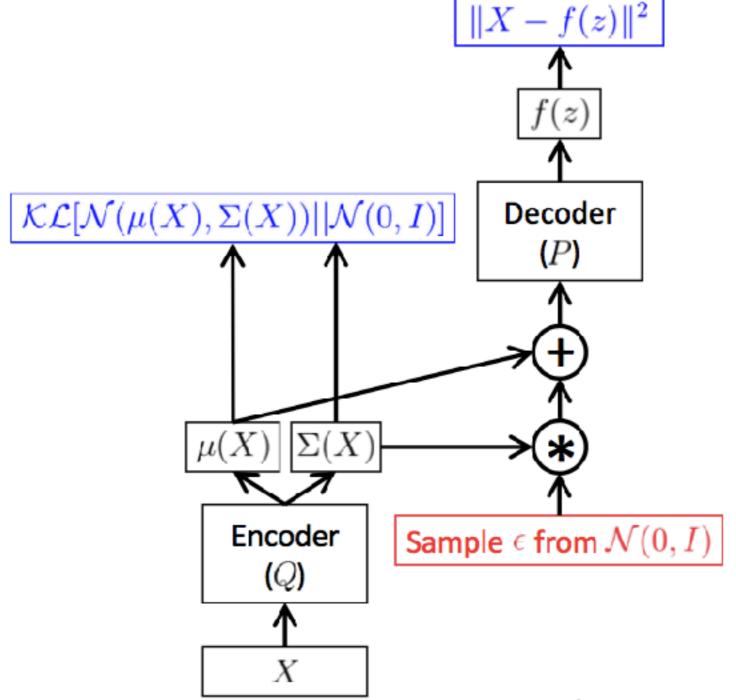


Figure Credit: Doersch (2016)

## An Example: Generating Sentences w/ Variational Autoencoders

# Generating from Language Models

 Remember: using ancestral sampling, we can generate from a normal language model

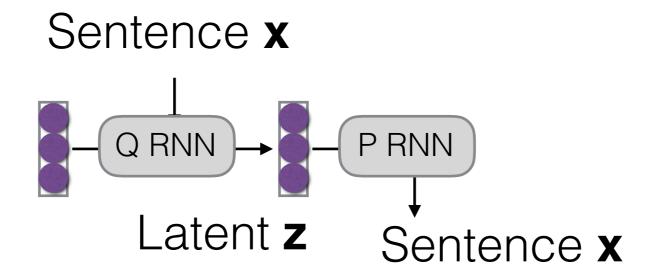
while 
$$x_{j-1} != "":  $x_j \sim P(x_j \mid x_1, ..., x_{j-1})$$$

We can also generate conditioned on something
 P(y|x) (e.g. translation, image captioning)

while 
$$y_{j-1} != "":  $y_j \sim P(y_j \mid X, y_1, ..., y_{j-1})$$$

## Generating Sentences from a Continuous Space (Bowman et al. 2015)

- The VAE-based approach is conditional language model that conditions on a latent variable z
- Like an encoder-decoder, but latent representation is latent variable, input and output are identical



# Motivation for Latent Variables

- Allows for a consistent latent space of sentences?
  - e.g. interpolation between two sentences

#### Standard encoder-decoder

i went to the store to buy some groceries .
i store to buy some groceries .
i were to buy any groceries .
horses are to buy any groceries .
horses are to buy any animal .
horses the favorite any animal .
horses the favorite favorite animal .
horses are my favorite animal .

#### **VAE**

```
"i want to talk to you."

"i want to be with you."

"i do n't want to be with you."

i do n't want to be with you.

she did n't want to be with him.

he was silent for a long moment.

he was silent for a moment.

it was quiet for a moment.

it was dark and cold.

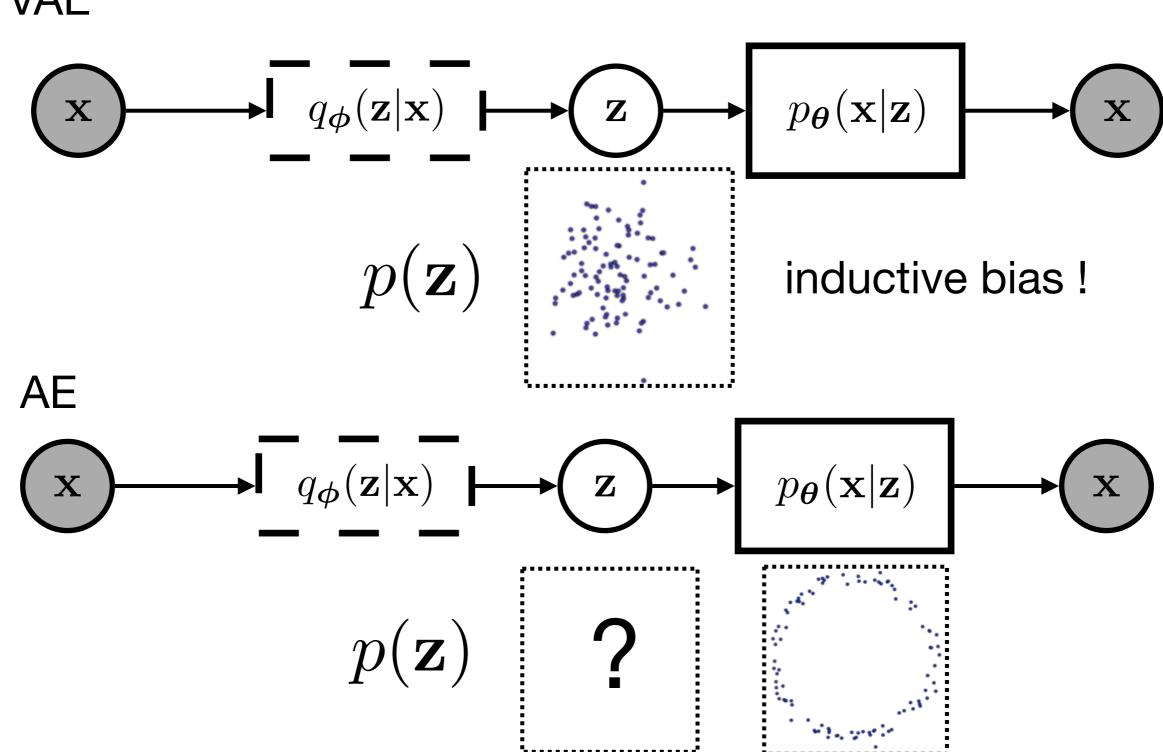
there was a pause.

it was my turn.
```

 More robust to noise? VAE can be viewed as standard model + regularization.

### VAE vs. AE

VAE



# Let's Try it Out! vae-lm.py

### Difficulties in Training

 Of the two components in the VAE objective, the KL divergence term is much easier to learn!

$$\mathbb{E}_{\boldsymbol{z} \sim Q(\boldsymbol{z} \mid \boldsymbol{x})}[\log P(\boldsymbol{x} \mid \boldsymbol{z})] - \mathcal{KL}[Q(\boldsymbol{z} \mid \boldsymbol{x}) || P(\boldsymbol{z})]$$

Requires good generative model

Just need to set the mean/variance of Q to be same as P

 Results in the model learning to rely solely on decoder and ignore latent variable

### Solution 1: KL Divergence Annealing

- Basic idea: Multiply KL term by a constant λ starting at zero, then gradually increase to 1
- Result: model can learn to use z before getting penalized

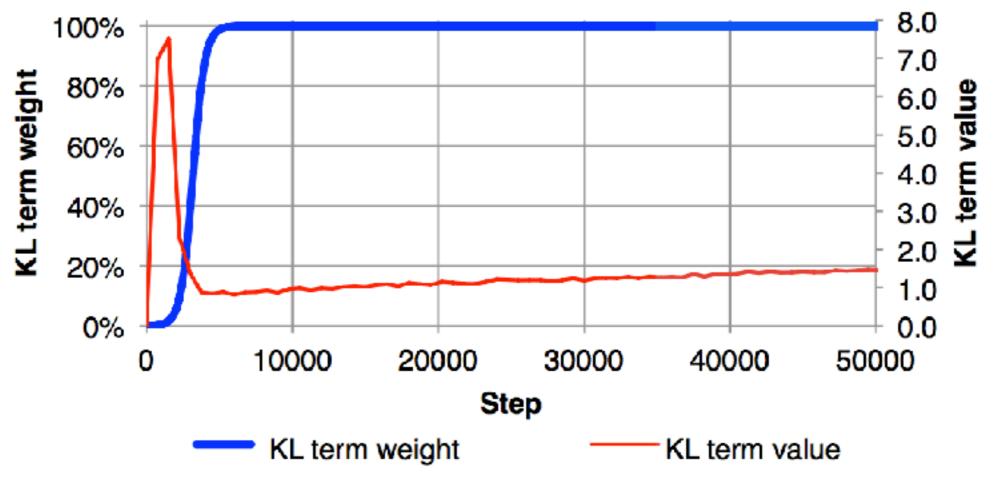
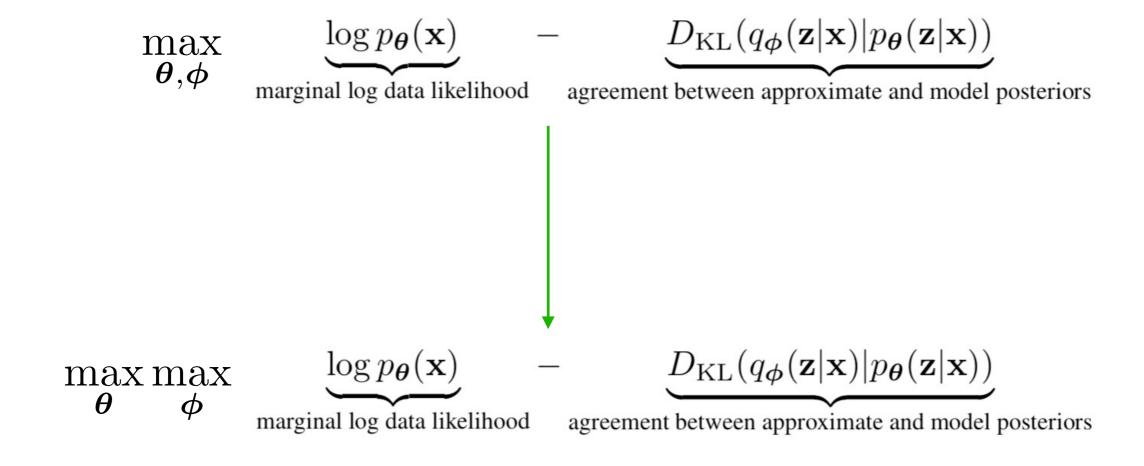


Figure Credit: Bowman et al. (2017)

# Solution 2: Weaken the Decoder

- But theoretically still problematic: it can be shown that the optimal strategy is to ignore z when it is not necessary (Chen et al. 2017)
- Solution: weaken decoder P(x|z) so using z is essential
  - Use word dropout to occasionally skip inputting previous word in x (Bowman et al. 2015)
  - Use a convolutional decoder w/ limited context (Yang et al. 2017)

#### Solution 3: Aggressive Inference Network Learning



(He et al. 2019)

# Handling Discrete Latent Variables

#### Discrete Latent Variables?

- Many variables are better treated as discrete
  - Part-of-speech of a word
  - Class of a question
  - Writer traits (left-handed or right-handed, etc.)
- How do we handle these?

### Method 1: Enumeration

For discrete variables, our integral is a sum

$$P(\boldsymbol{x}; \theta) = \sum_{\boldsymbol{z}} P(\boldsymbol{x} \mid \boldsymbol{z}; \theta) P(\boldsymbol{z})$$

 If the number of possible configurations for z is small, we can just sum over all of them

#### Method 2: Sampling

- Randomly sample a subset of configurations of z
  and optimize with respect to this subset
- Various flavors:
  - Minimum risk training
  - Maximize ELBO loss
- Score function gradient estimator Policy Gradient Method
- Unbiased estimator but high variance need to control variance

### Method 3: Reparameterization

(Maddison et al. 2017, Jang et al. 2017)

Reparameterization also possible for discrete variables!

#### **Original Categorical Sampling Method:**

$$\hat{\boldsymbol{z}} = \text{cat-sample}(P(\boldsymbol{z} \mid \boldsymbol{x}))$$

#### **Reparameterized Method**

$$\hat{z} = \operatorname{argmax}(\log P(z \mid x) + \operatorname{Gumbel}(0,1))$$
 where the Gumbel distribution is  $\operatorname{Gumbel}(0,1) = -\log(-\log(\operatorname{Uniform}(0,1)))$ 

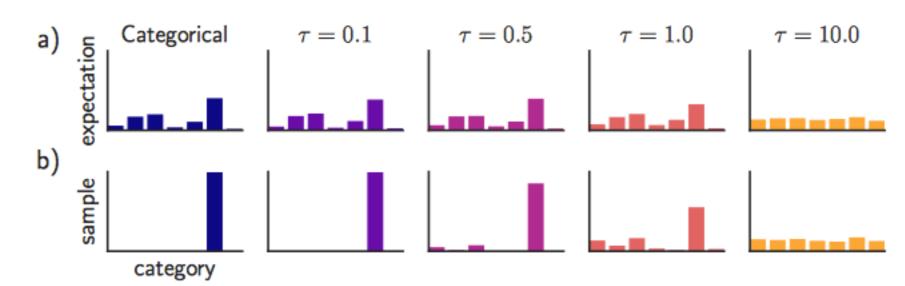
Backprop is still not possible, due to argmax

### Gumbel-Softmax

- A way to soften the decision and allow for continuous gradients
- Instead of argmax, take softmax with temperature τ

$$\hat{\boldsymbol{z}} = \operatorname{softmax}((\log P(\boldsymbol{z} \mid \boldsymbol{x}) + \operatorname{Gumbel}(0,1))^{1/\tau})$$

As τ approaches 0, will approach max



# Application Examples in NLP

# Variational Models of Language Processing (Miao et al. 2016)

 Present models with random variables for document modeling and question-answer pair selection

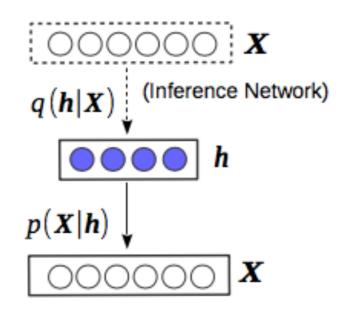


Figure 1. NVDM for document modelling.

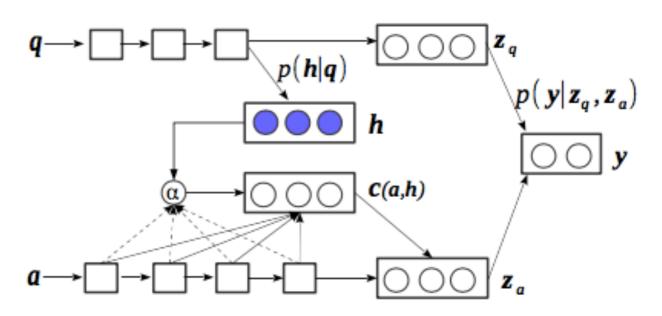


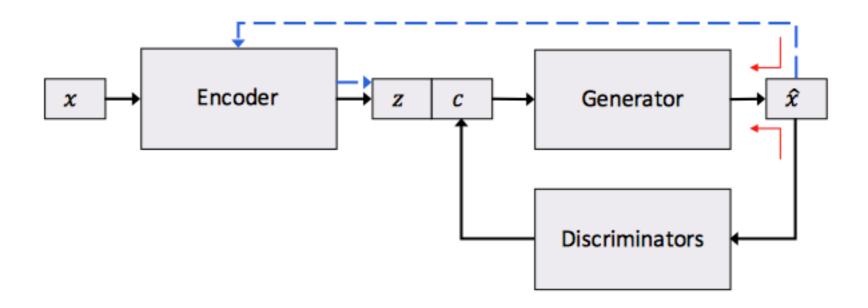
Figure 2. NASM for question answer selection.

 Why random variables? Documents: more consistent space, question-answer more regularization?

### Controllable Text Generation

(Hu et al. 2017)

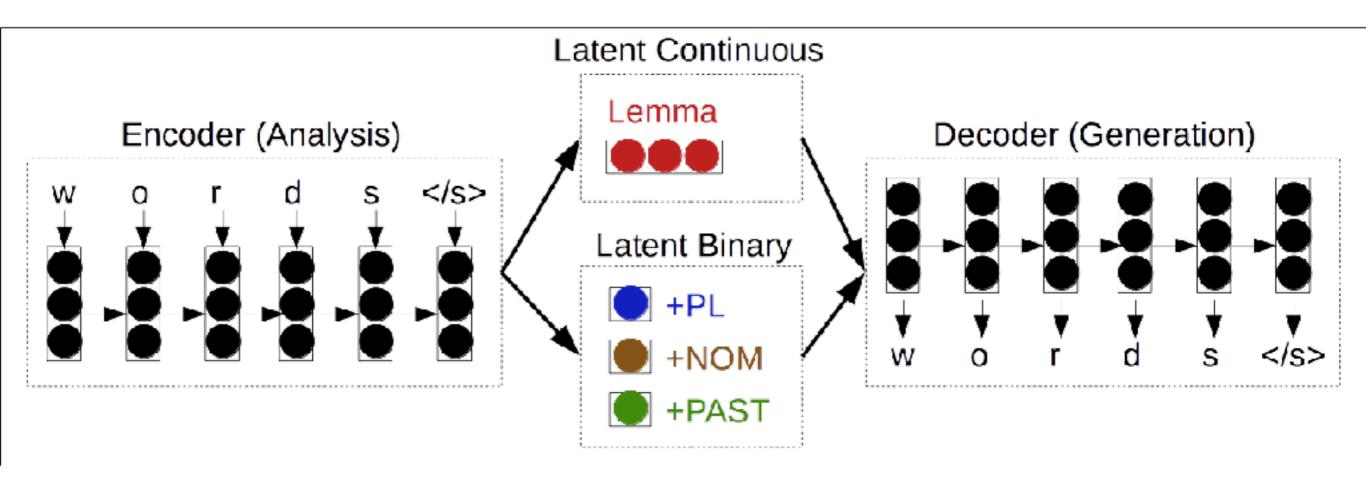
 Creates a latent code z for content, and another latent code c for various aspects that we would like to control (e.g. sentiment)



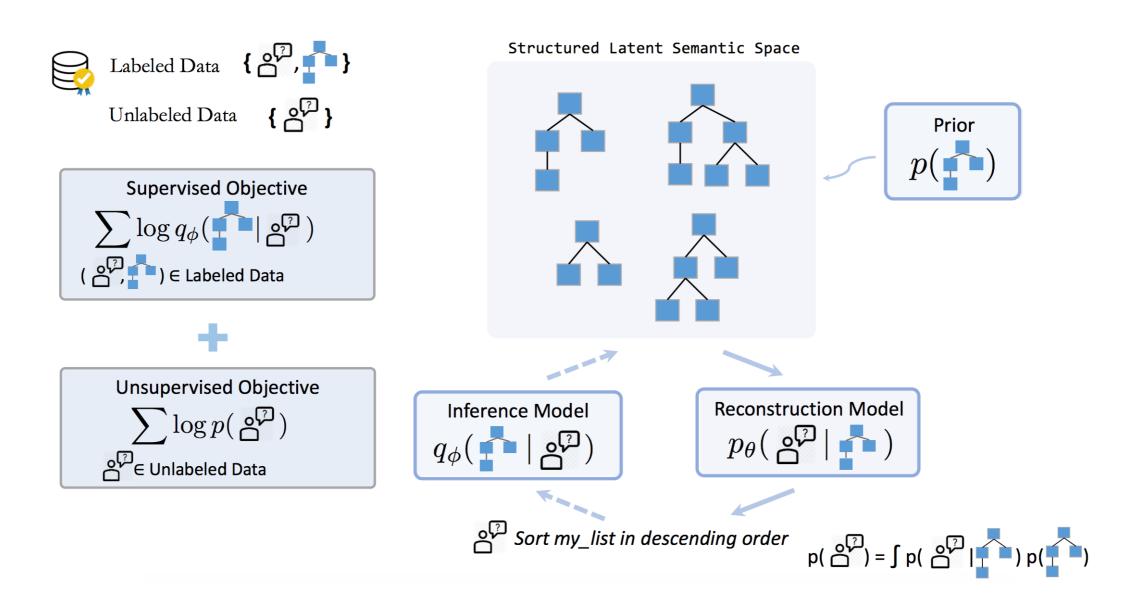
Both z and c are continuous variables

## Controllable Sequence-to-sequence (Zhou and Neubig 2017)

 Latent continuous and discrete variables can be trained using auto-encoding or encoder-decoder objective

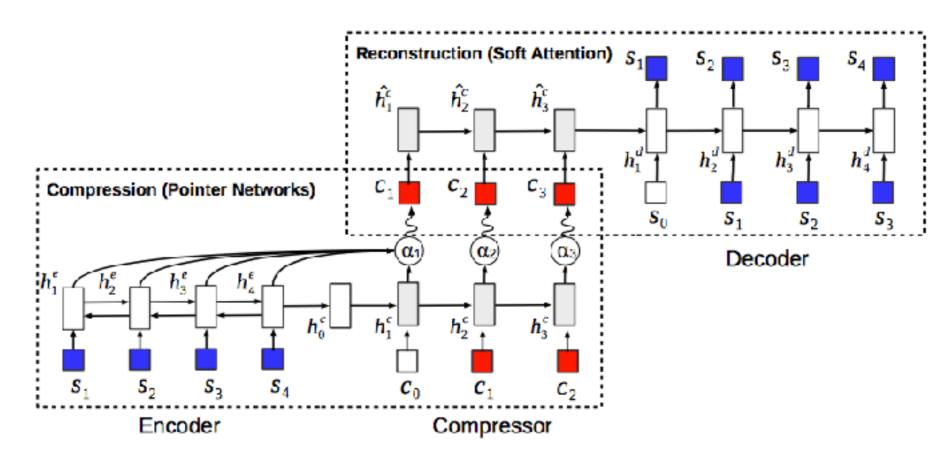


### STRUCTVAE: Tree-structured Latent Variable Models for Semi-supervised Semantic Parsing (Yin et al. 2018)



# Symbol Sequence Latent Variables (Miao and Blunsom 2016)

Encoder-decoder with a sequence of latent symbols



- Summarization in Miao and Blunsom (2016)
- Attempts to "discover" language (e.g. Havrylov and Titov 2017)
  - But things may not be so simple! (Kottur et al. 2017)

### Questions?